Processing Algorithm for obtaining the Impulse Response in a MIMO Ultrasonic System

A. Ochoa, J. Ureña, A. Hernández, M. Mazo, C. De Marziani, M. C. Pérez
University of Alcalá/Electronic Department, E-28806 Alcalá de Henares (Madrid), Spain
E-mail: {alberto, urena, alvaro, mazo, marziani, carmen}@depeca.uah.es

Abstract—In this work a processing algorithm is presented to obtain the impulse response of the transmission channel in a MIMO ultrasonic system. In this algorithm the macro-sequences (Mₘ) obtained from complementary sets of M sequences are used to obtain the impulse response of the environment. The signals received by the sensorial system are demodulated and correlated to find the transmitted signals. It is possible to obtain more detailed information from the environment through the analysis carried out over the correlated signal to extract the impulse response of the environment. In order to eliminate the cross correlation among the Mₘ and improve the localization of the correlation peaks, a post-process algorithm is applied to the correlated signal to find them with more accurately. Finally, a matrix of impulse responses of the transmission channels can be recovered by using the correlation peaks.

I. INTRODUCTION

The detection and classification in ultrasonic systems (sonar) have changed every year with the purpose of increasing the percentage of success in the classification. It implies an increase in the processing capacity in the emission stage and in the reception, and the use of more complex algorithms. In these systems, the number of transducers in the sensor has been increased; and, at the same time, different geometric configurations of the transducers have been tested in order to obtain more detailed data of the environment in every scanning process [1].

The use of multiple transducers in the emission and reception stages involves a change in the traditional model of the system, and this implies the use of a technique that provides detailed information for all the physical links appearing among emitters and receivers. The MIMO (Multiple-Input Multiple-Output) systems are often used here due to their great capacity and diversity to analyze the effect of several types of signals interfering in a transmission system [2]. Since a system has a simultaneous transmission and reception, where several emitters and receivers are involved, the transference function between every emitter and every receiver should be obtained. A complete model of the MIMO transmission system can be stated with all the generated functions.

Depending on the environment, a signal emitted by an ultrasonic transducer Tₑmi arrives at the receiver Tᵣce through different ways, so a Multipath propagation model should be used [3]. There are different techniques to determine the impulse response h[k] of a transmission channel [3] [4]; for example, the excitation of a system with a short pulse or an encoded signal and the subsequent auto-correlation to find the signal transmitted in the echo received. Regarding the techniques used for the correlation of the received signal, it is common to encode with pseudo-random sequences [5] or complementary sequences [6].

In this work a processing algorithm is presented to estimate the impulse response in a MIMO ultrasonic system, by using Macro-Sequence obtained from the Complementary Sets of M Sequences (M-CSS) to take advantage of their auto-correlation properties [6]. The system is composed by $\mu$ transducers, which simultaneously emit a signal encoded with a particular and different macro-sequence. Simultaneously, at each receiver, the echoes are received and processed to obtain their corresponding responses, which are related to each emission and to the physical transmission channel between the considered emitter and receiver. The goal of this paper is provide a MIMO model of a sensor system mounted on a mobile robot with the aim of obtaining more information from the sensor in every scanning process about a changing environment.

The rest of this paper is organized as follows. Section II, explains how the macro-sequences are obtained from complementary sequences. In Section III, the ultrasonic transmission system and its MIMO model are analyzed. In Section IV, the algorithm to estimate the impulse response is developed. In Section V, some simulations are shown and; finally, conclusions are discussed in Section VI.

II. MACRO-SEQUENCES OBTAINED FROM THE COMPLEMENTARY SET OF M-SEQUENCES

A Complementary Set of M-Sequences (M-CSS) is a set of binary sequences with length $L$, where every sequence of the set contains either $+1$ or $-1$. The principal characteristic of these sets is that the addition of their aperiodic auto-correlation functions is $ML$ for zero-shift and null for all non-zero shifts [4] [6]. In a particular form, if a 32-CSS of 32 sequences $(M=32)$ $\{a_1,a_2,\ldots,a_{32-1},a_{32}\}$ with length $L = 32$ is used, the sum $\Phi[k]$ of the auto-correlation functions $\phi_\alpha[k]$ is:

$$\Phi[k] = \sum_{\alpha=1}^{M} \phi_\alpha[k] = \begin{cases} M \cdot L_0 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

(1)

A set M-CSS of length $L$ can be obtained using an Efficient Set of Sequences Generator (ESSG) and, in the same form, an Efficient Set of Sequences Correlator (ESSC) can be implemented [4] [6]. Both algorithms were obtained to generate and correlate these sets of sequences, respectively, and their main purpose is to reduce the number of operations to be carried out. Every set is
generated from a seed $W$, of binary values \{+1, -1\} that has a decimal representation given by $p$ [4][6], which is characteristic of every set. In order to obtain the encoded signal used to excite every emitter, the bits of the $M$ sequences from the $M$-CSS should be interleaved to generate a Macro-Sequence ($M_\mu$) of length $L_{MS} \cdot M \cdot L$. In (2) the generation of a macro-sequence $M_\mu$ is shown, by using every bit of the sequences obtained from seed $p$ [4].

$$M_\mu[k] = [x_{1,1} \ x_{1,2} \ldots \ x_{1,L} \ x_{2,1} \ x_{2,2} \ldots \ x_{2,L} \ldots \ x_{n,1} \ x_{n,2} \ldots \ x_{n,L}]$$

Where $x_{n,m}$ is every bit of the sequences of set, $n \in \{1,2,\ldots,M\}$ and $m \in \{1,2,\ldots,L\}$.

In Fig. 1 the auto-correlation $\Phi_{\mu}$ of a $M_\mu$ obtained with the interleaving of bits of the 32-CSS is shown. The length of the $M_\mu$ is 1024 ($M=32$, $L=32$) and it was generated with a seed $p=8$.

Figure 1. Auto-correlation $\Phi_{\mu}$ of a $M_\mu$ with length 1024, obtained from a 32-CSS ($M=32$) with interleaving.

In order to increase the energy transmitted for an ultrasonic transducer a digital modulations can be used to emit the macro-sequence $M_\mu$, such as binary PAM or BPSK [7]. With this modulation the signal is emitted in a narrow bandwidth transducer.

### III. MODEL OF TRANSMISSION IN AN ULTRASONIC SYSTEM

Because the impulse response of a transmission channel contains information not only from the environment but also from the reflector that generates the echo, this information can be used for the classification process of the reflectors. In order to analyze the ultrasonic sensorial system (emission-reflection-reception), a MIMO model of the system can be used, since a complete analysis of the system can be considered at different instants of time.

#### A. Sensorial System

In Fig. 2 the geometrical structure of the ultrasonic sensorial system with 4 transducers ($\mu=4$) and the localization of an ultrasonic reflector in front of the structure is shown. This physical structure will be used in all the simulations for estimate the impulse response.

Figure 2. Structure of an ultrasonic system (4 emitters/receivers).

#### B. MIMO Model of The Ultrasonic System

In general terms, the relation between the input and output (MIMO system model [2]) of the transmission system with $\mu$ emitters/receivers (see Fig. 2), can be defined by (3). The signal received by the receiver $j$ can contain $\mu$ signals involved that correspond with the echoes generated by the transmission of the macro-sequences by the emitters.

$$\psi[k] = \sum_{l=0}^{L_{MS}} H_{ij} (z^{-l}) \psi_{\mu}^{\mu\mu} + \eta[k]$$

Where the signal $\psi[k]=[y_1[k] \ y_2[k] \ldots \ y_{\mu}[k]]^T_{(\mu\lambda)}$ is a vector containing the signals received by every transducer; $\psi_{\mu}^{\mu\mu}=[m_{11}[k] \ m_{21}[k] \ldots \ m_{\mu1}[k]]^T_{(\mu\lambda)}$ is a vector containing the Macro-Sequences transmitted by the transducers; $\lambda$ is the order of transmission channel; $\{H_{ij}\}_{i,j=1,\ldots,\mu}$ are the matrix of dimension $(\mu \times \mu)$ that contain the impulse response value of every transmission channel at the instant $l$, and $\eta[k]=[\eta_{1}[k] \ \eta_{2}[k] \ldots \ \eta_{\mu}[k]]^T_{(\lambda\lambda)}$ is the noise vector, where each element $\eta_{i}[k]$ is associated with a transducer $i$. In (4) the matrix $H_{ij}$ is shown, which contains the impulse response $h_{ij}$ of the physical links between an emitter $i$ and a receiver $j$ at the instant $l$.

$$H_{ij} = \begin{bmatrix}
    h_{ij,1} & \ldots & h_{ij,\mu}
    
    \vdots & \ddots & \vdots 
    
    h_{ij,1} & \ldots & h_{ij,\mu}
\end{bmatrix}$$

In (5) the model of the system as the addition of finite impulse responses (FIR) is defined, where the matrix $H_{ij}(z^{-1})$ contains the sum of the $H_{ij}$ matrices for any instant $l$ considered by the system. This matrix represents the attenuation and change in phases for the signal received by the transducers with a delay $l$ in $z$-domain.

$$H_{ij}(z^{-1}) = \sum_{l=0}^{L_{MS}} H_{ij} \cdot z^{-l}$$
by emitter $i$ through the auto-correlation ($\otimes$). After obtaining the correlated signal $\Phi_i$, a threshold is applied to obtain the impulse response $h_{ii}$ of the transmission channel between an emitter $i$ and a receiver $j$.

$$\Phi_i[k] = \psi \left[ \frac{k - j}{\mu} \right] \otimes M_x[k], \quad \forall |i|, |j| = 1, 2, \ldots, M$$

(6)

B. Post-processing algorithm of the correlated signal

With the purpose of locating with more accuracy the correlation maximums and reducing the noise caused by the cross-correlation of the macro-sequences, a technique, called correction factor or post-process, can be applied [8]. Whenever it is applied, this correction technique can eliminated the noise added to the signal during its transmission through the environment and the noise generated by the cross-correlation among the macro-sequences simultaneously received.

This technique consists of determining the behavior of the correlated signal without any kind of disturbance, so it can be previously calculated as reference $\Phi_{corr}[k]$. After, the energy of the received signal $\Phi_i[k]$ (non-ideal) is analyzed through a window of length $2w-1$ at instant $k$, where the length of the reference signal $\Phi_{corr}[k]$ is equal to $2w-1$. In (7) this processing is shown, where the signal $\Phi_i[k]$ is the new output of the system.

$$\Phi_i'[k] = \text{sign}(\Phi_i[k]) \cdot \left\{ \sum_{n=1}^{k=2w-1} \left| \Phi_i[n] - \frac{\Phi_i[k]}{\Phi_{corr}[k]} \cdot \Phi_{corr}[n] \right| \right\}$$

(7)

If the energy existing in the analysis window open in $\Phi_i$ at instant $k$ is equal to the energy of the reference signal $\Phi_{corr}[k]$ in a window of the same dimension, the post-processing algorithm gives the value of the energy sum to the new signal $\Phi_i'$ generated at instant $k$, so the correlation peaks are increased. Otherwise, if the energy is not equal, the correction factor algorithm gives a determined minimum value to $\Phi_i'$ at instant $k$.

V. SIMULATIONS

If a delta is applied to the emission block in the system described in Section III, 4 ($\mu=4$) $M_S$ are generated to excite the emitters of the transmission system and afterwards to compute the impulse response of the physical links in the environment. In (8) a set of impulse responses associated to the matrix $H_i$ is shown, which represent the behavior of the transmission channels in a digital MIMO system during the transmission of the macro-sequences through the environment. The matrix $H_i$ supplies detailed information of the behavior of the environment and furthermore, the geometrical position of the ultrasonic reflector in front of the sensor is given too.

$$H_i(z^{-1}) = \begin{bmatrix} z^8 & z^{-8} & z^{-8} & z^{-8} \\ z^{-6} & z^{-9} & z^{-9} & z^{-9} \\ z^{-10} & z^{-11} & z^{-12} & z^{-13} \\ z^{-14} & z^{-15} & z^{-15} & z^{-15} \end{bmatrix}$$

(8)

Every 1024-bit $M_5^0$ is obtained from a 32-CSS of 32 sequences ($M=32$) with length $L=32$. In Fig. 4 the result of the identification of the $M_5^0$ ($p=5$) transmitted by the emitter E/R$_1$ and received by the transducer E/R$_3$ is shown. Moreover it can be observed the result of applying the post-processing algorithm to the correlated signal to determine more accurately the chosen peaks and to recover the matrix $H_i$. During the transmission/reception process performed by the system, the transducer E/R$_1$ emits its $M_5^0$ and, simultaneously, the other three $M_5^0$ are transmitted by the rest of transducers. In the reception processing, every transducer receives the echoes of the signal transmitted by itself and by the other transducers. The system can identify each one of the $M_5^0$ and assign to every $\Phi_i[k]$ their corresponding correlation, even if the signals are received simultaneously by the transducers.

![Figure 4. Results of the identification of the macro-sequence $M_5^0$](image-url)

The previous simulations were carried out without including the model of the transducer, and for this reason the value of correlation peaks of the $M_5^0$ corresponds with the parameters expected for theirs amplitudes and delays. In (9) the matrix $H_i^*$ is presented, which is estimated with the correlation and threshold algorithm to obtain the amplitude of the impulse response applied. The delays of every impulse response are calculated with the post-processing algorithm. This estimated matrix is practically the same as the one applied to represent the physical links, and there is only a minimum error in the amplitude of the considered impulse responses.
A new matrix $H_s$ with a different group of impulse responses, is used to emulate the behavior of the environment in the transmission system, where some $h_{ik}[k]$ represents the phenomenon of multipath. The matrix $H_\ell$ of impulse responses applied to the environment is shown in (10), where $h_{ik}[k]$ is the impulse response representing the multipath by a double reflection.

$$H_\ell(z^{-1}) = \begin{bmatrix} 0.99z^{-1} & 1.10z^{-4} & 1.09z^{-8} & 1.10z^{-12} \\ 0.99z^{-1} & 1.07z^{-9} & 1.00z^{-13} & 1.00z^{-15} \\ 1.10z^{-10} & 1.10z^{-11} & 1.09z^{-12} & 1.09z^{-13} \\ 1.01z^{-4} & 0.99z^{-13} & 1.00z^{-14} & 1.00z^{-15} \end{bmatrix}$$

The estimated impulse responses applied to the environment is different from the original matrix $H_\ell$ estimated is different from the original matrix $H_\ell$ applied to the system only in amplitude of their delays, as is observed in (11).

$$H_\ell'(z^{-1}) = \begin{bmatrix} z^2 & z^4 & z^6 & 1.2z^8 \\ 0.9z^{-6} & 0.9z^{-9} & z^{-8} & z^{-4} \\ 0.9z^{-11} & z^{-11} & 1.1z^{-12} & z^{-11} \\ z^{-4} + 0.8z^{-16} & z^{-13} & z^{-14} & z^{-15} \end{bmatrix}$$

Fig. 5 shows some results obtained after the identification of the impulse responses $h_{ik}[k]$ applied to the environment by the emission of the $M_S^{th}$ transmitted by E/R4. The correct identification of the $M_S^{th}$ emitted by E/R4 is detected over the echo received by the transducer E/R4. Moreover in the correlated signal, two deltas are shown in (10), where $h_{ik}[k]$ is the impulse response given by $h_{ik}[k]$. In order to verify the estimation algorithms of the impulse responses, the model of transducers is included in this simulation. The matrix $H_\ell'$ estimated is different from the original matrix $H_\ell$ applied to the system only in amplitude of their impulse responses but not in their delays, as is observed in (11).

$$H_\ell'(z^{-1}) = \begin{bmatrix} 1.03z^{-2} & 0.81z^{-4} & 1.22z^{-6} & 1.18z^{-8} \\ 0.87z^{-6} & 0.8z^{-9} & 1.04z^{-10} & 0.98z^{-13} \\ 1.00z^{-10} & 0.95z^{-11} & 1.14z^{-12} & 1.02z^{-13} \\ 1.04z^{-10} + 0.71z^{-16} & 0.9z^{-13} & 1.17z^{-14} & 0.95z^{-15} \end{bmatrix}$$

VI. CONCLUSIONS

In this work the analysis of a MIMO ultrasonic system to estimate the impulse response of the transmission channels is considered. Through the analysis carried out on the ultrasonic system, it is possible to obtain more information from the environment by means of the designed sensorial structure. The macro-sequences used in the simulator are constructed from the complementary set of $M$ sequences to take advantage of their characteristics, and each macro-sequence is used to excite an emitter of the sensorial system.

As the ultrasonic system is composed by 4 ($\mu=4$) transducers, 4 $M_S$ are generated for their transmission. In every reading process up to 16 signals are received and, with the correlation and threshold algorithm applied to the echoes up to 16 impulse responses are obtained. To locate more accurately the correlation maximums, a post-processing algorithm is applied over the signals $q_j$ to reduce the noise that was added to these signals when were transmitted by the environment. Moreover a good estimation of the impulse responses of the physical links through the estimated matrix $H_\ell'$ is obtained, which shows the behavior of the transmission system during a perturbation. As future works, the information obtained from the impulse responses of the transmission channels can be used as classification parameter for recognition of complex reflectors.

ACKNOWLEDGMENTS

This work has been possible thanks to the Spanish Ministry of Science and Technology through the PARMEI project (ref. DIP2003-08715-C02-01), and to the Comunidad de Madrid with ANESUS project (ref. CAM-UAH2005/016).

REFERENCES


