Relative Positioning System Using Simultaneous Round Trip Time of Flight Measurements

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Abstract— The determination of the relative position among mobile objects or members of a robot team is a useful information when the systems are not restricted to a particular environment and they cannot depend on an external infrastructure to obtain their location information. The solution for this kind of problems cannot be divided and treated separately for each object. Therefore it is necessary to implement mechanisms that allow to simultaneously determine the spatial relations among objects not knowing their absolute location in the environment. After that, a positioning algorithm can be computed whose complexity depends on the number of observations made and the precision required in the system. In this work, the implementation of a relative positioning system is described where only acoustic emissions are made in order to obtain information about the other objects. These data are obtained by using simultaneous round-trip-time-of-flight measurements. Finally, using this information, the object positions are computed with a multidimensional scaling technique, and a closer solution is achieved with least-square algorithms.

I. INTRODUCTION

The relative position among mobile objects is an important information when the systems are not restricted to a particular environment [1], and cannot depend on any external infrastructure as in Local Positioning Systems (LPS) [2]. In this case, the information is collected between pairs of objects, usually called ‘Relative Location’ [3]. The objects must have the hardware that allows to interact one to each other. As a result, it is necessary to design a sensor network that allows to obtain the spatial relationships and to communicate the data collected by each one before computing their relative positions [4].

The position estimation in these kinds of systems is more difficult than in LPS system, because the positions are determined by using measurements among objects whose location is not known in the environment. One of the more used relative positioning algorithm is the Multidimensional Scaling Technique (MDS) [5], metric or classic. This algorithm allows to obtain a geometric configuration of the positions among objects, in the smallest number of dimensions, only using the distances measured between the objects. Nevertheless, the data collected in the sensor network, are usually influenced by different parameters that lead to errors in the coordinate estimation. In order to reduce the error in the MDS solution, least-square methods can be used [5] [6] [7].

In this work, the information obtained with a low complex ranging method, called Simultaneous Round Trip-Time-of-Flight (S-RTOF) [8], is used to determine the relative positions. This mechanism makes possible to simultaneously determine temporal relations among the emissions made from every node by only using acoustic signals. This fact allows to remove all type of additional synchronization connections by RF or IR, reducing the hardware complexity. Besides, in indoor spaces, the RF signals can be interfered by other systems such as 802.11 networks, or by spurious signals from the fluorescent illumination in the case of IR.

Due to the features of the S-RTOF mechanism, it is necessary to analyze the signal processing delays that corrupt the measurement of time relations, also called pseudo-time-of-flight (pTOF). Using these information, the positions are computed by the MDS technique and, considering the different error sources, a close solution is obtained by performing Levenberg-Marquardt (LVM) algorithm.

The paper is organized as follows: next Section shows the relative positioning system architecture. In Section III the S-RTOF is described considering the different parameters involved in the measurement. Section IV depicts the positioning algorithms, using a first approach estimation by MDS technique and its optimization with the Levenberg-Marquardt algorithm. Finally simulations and conclusions about the proposed algorithm are shown in Sections V and VI respectively.

II. LOCALIZATION SYSTEM ARCHITECTURE

The basic architecture of the system is shown in Fig.1, where every object carries a node with acoustic transducers as sensor technology. The main characteristics of this system are: 1) the smallest external infrastructure as possible is used; 2) local computing of the relative positions with all the objects; 3) all nodes are equal in their architecture and functionality; 4) no physical connection among them; 5) only using acoustics emissions, reason why RF or IR connections will not be required for synchronization. Acoustic transducers are used due to their low cost, easy implementation and availability in mobile computer systems. About drawbacks, environmental aspects affect their performance outdoors, but in indoor spaces, these are minimized.
environment at different times, according to their distribution on the
in every slave node by their microphones
node. This emission, called
At this moment, the measurement of
acoustic signal with a particular encoding (see Fig. 2.a).
for the real-time processing of the acoustic emissions [9].
reduction in hardware complexity and computational load
suitable correlation properties and allow to obtain a high
Complementary Sets of Sequences (CSS) that have
used in order to simultaneously detect the emissions made
by every object. The emissions are encoded by
for the object positions.
functionality, reason why every node can locally compute
the emissions made
Because all nodes are equal in their architecture and
round-trip-time-of-flight measurement of acoustic
systems is the measurement, in a simultaneous way, of all
the spatial relations among objects. In order to obtain
these, a low-complex method, based on simultaneous
wave equations, is possible to
Also, a non-centralized system is obtained
because all nodes are equal in their architecture and
functionality, reason why every node can locally compute
the object positions.
A Code Division Multiple Access scheme (CDMA) is
used in order to simultaneously detect the emissions made
by every object. The emissions are encoded by
Carrier Sense Multiple Access (CSMA) is used
"Master"
process. In order to determine which one starts, called
"Master Request"
starts in this
, is detected
"Master"
node emits its
emission, has been developed [8].
An important capability of the relative localization
systems is the measurement, in a simultaneous way, of all
the spatial relations among objects. In order to obtain
these, a low-complex method, based on simultaneous
round-trip-time-of-flight measurement of acoustic
emission, has been developed [8].
A. Principle of measurement
Because all nodes are equal in their architecture and
functionality, anyone of them can start the location
process. In order to determine which one starts, called
"Master" node, a multiple access technique such as
Carrier Sense Multiple Access (CSMA) is used. Let
assume that the Master node has a position with
cartesian coordinates \((x, y, z)\), described by vector \(p_{\text{Master}}\), and
positions of the slave nodes \(q\) and \(l\) are given by vectors \(p_q\) and \(p_l\). At a given instant, the "Master" node emits its
acoustic signal with a particular encoding (see Fig. 2.a).
At this moment, the measurement of RTOF starts in this
node. This emission, called "Master Request", is detected
in every slave node by their microphones Mic-\(q\) and Mic-\(l\)
at different times, according to their distribution on the
environment
In response to "Master Request", every node emits its
characteristic code denoted as "Ack. Node", which travels
towards the "Master" node and also to the other slave
nodes (see Fig 2.b). In this way, in the "Master" node, the
time from the emission of the "Master Request" until the
detection of every "Ack. Node" can be computed. Also,
taking advantage of the slave node emission, it is possible
to compute temporal relations, from the 'Master Request'
detection, among their emissions in every slave node.
B. Pseudo-Time-of-Flight Equations
According to the described algorithm, it is possible to
obtain temporal relations among the emissions of the
different nodes, also called pseudo-times of flight (pTOF)
[10], since it is not a direct measurement of the time-of-
flight of the acoustic emissions. These measurements are
also affected by the delays on the signal processing in
every node.
In the case of the pTOFs measured between Master and
every slave node, it is necessary to consider the delay until
the signal is processed and emitted in the speaker, denoted as \(t_{\text{Spk}}\). Also, in the reception stage there exists a delay \(t_{\text{Mic}}\)
from the capture of the signal by the microphone up to the
hardware processing. The following equation summarizes
these considerations:
In (2), taking into account that the speeds of the nodes
are smaller than the propagation speed of the acoustic
waves, it is possible to obtain (2):
\[
i_{\text{Master}-q} = 2 \left( \frac{\|p_{\text{Master}} - p_q\|}{c} + t_{\text{Spk}} + t_{\text{Mic}} \right) + T_{\text{CODE}}
\]
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\]
Where \(t_{\text{Spk}} = t_{\text{Spk}} + t_{\text{Mic}}\) is the time required by nodes to
process the signals in the emission and reception blocks.
On the other hand, for example, in the slave node \(q\)
after the detection of Master Request, it is possible to
measure the following pTOF with the 'Ack. Node 1'.

![Figure 1. General scheme of the proposed relative positioning system.](image)

![Figure 2. a) Emission of the request from the "Master" to start the location process. b) Acknowledgement from every slave node with the aim of measuring the pTOF.](image)
Using (3) the following relation among the emissions involved can be obtained:

\[ t_{\text{qMic}} - t_{\text{Master}} = t_{\text{q}} + t_{\text{qMic}} + T_{\text{CODE}}. \]  

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Where \( t_{\text{q}} \) is the pTOF measured in the slave node q with the slave node \( i \).

IV. POSITIONING ALGORITHM

The algorithm used to determine the relative positions among nodes is shown in Fig. 3. After executing the S-RTOF ranging method and assuming that the nodes communicate the pTOF computed in each one of them, the position estimations can be performed. A first approximation by MDS algorithm is made, and then, using a non-linear square method, results can be optimized.

A. MDS Estimation of coordinates

Before starting the position estimation, the distances among nodes are computed (Step 1 in Fig.4) by (2), (4) and using the measured pTOF. Also the signal processing delays are roughly known in every node by auto-calibration or off-line characterization.

The second step is to build a matrix \( B^* \), called dot-product, that considers the distances among objects from a reference point, being the most suitable the centroid (midpoint) of the figure formed by the objects in a two or three dimension system (2D or 3D) [7]. The following step is the decomposition in singular values (SVD) of \( B^* \). Considering the properties of the resulting eigenvectors and eigenvalues matrices, U and S, the coordinates of the objects can be obtained by selecting the first two or three columns of matrix \( X^* \) according to the dimensions of the system (Step 3 on Fig.4). In order to fit the obtained results to the reference system of every node, it is necessary to consider a rotation and translation process by means of the transformation matrix \( T \), because the obtained results are referred to the midpoint of the objects. The matrix \( T \) is built by using the object, where the calculations are made, as the coordinate origin and by selecting another one that forms a line with the previous one in the case of a 2D system.

B. Optimization Results

Assuming that the measurements by S-RTOF method are independently contaminated by Gaussian noise, with null mean and variance \( \sigma^2 \), the estimation of coordinates consists of solving a non-linear equation system [6][5][7]. A suitable method to solve this kind of problems is the Levenberg-Marquardt algorithm, which allows to find a local minimum using a particular start point. Thus, the results obtained with the MDS algorithm are a good starting point to estimate the coordinates by the LVM method.

With the aim of obtaining a closer solution, the unknown coordinates and the signal processing time should be considered. However, the estimation of all the signal processing times increases the complexity of the equation system to be solved. In order to simplify the problem, the processing times can be considered similar in nodes, reducing the number parameters to be estimated (5).

\[
DQ + 1 - D(D+1) \geq 2 \quad (5)
\]

Where \( D \) is the number of dimensions of the coordinate system; and \( Q \) is the maximum number of elements.

The first term in (5) is the number of unknown coordinates. Also, it is necessary to add the signal processing delay unknown. Finally, the last term is the number of coordinates required to be known in order to obtain a minimum reference system. In order to solve the equation system, the number of observations made in the system should be higher than (5). Because the number of observations linearly independent is \( Q(Q-1)/2 \), the minimum number of nodes required to determine the positions is:

\[
\frac{Q(Q-1)}{2} \geq DQ + 1 - D(D+1) \quad (6)
\]

According to (6), if the positions are computed in a 2D, the minimum number of nodes required is \( Q=4 \); in the case of 3D coordinate system, it is necessary to have \( Q=6 \) nodes in the system.
V. SIMULATIONS AND RESULTS

In order to compare the performance of the positioning algorithms, a Monte-Carlo simulation has been performed. A topology of eight nodes, distributed in 2D as shown in Fig. 5 is used, where every node has a pTOF vector \( \mathbf{T} \) whose elements are corrupted with additive Gaussian noise with null mean and variance \( \sigma^2 \). For a given value \( \sigma^2 \), the characterization is performed for one thousand tests. On the proposed topology, the Master Node condition is assigned to the object \( M_b \) with coordinates \((0,0)\), whereas the node \( M_b \) forms a line with coordinates \((0, y_b)\), obtaining the reference system. The LVM optimization was performed with the Matlab® function levenqnl.

Figure 5 shows the 95% uncertainty ellipses (2\( \pi \)) obtained from the coordinate estimation when the pTOFs have a Gaussian noise of variance \( \sigma^2 = 100 \mu s^2 \). Using the LVM optimization it is possible to obtain a closer solution on the estimation of coordinates as shown with dotted lines in Fig. 5.

Computing the Monte-Carlo simulations over different variance values of Gaussian noise, it is possible to obtain a comparative of the variance and the bias error on the position estimation with the two proposed methods. The variance obtained with the LVM optimization is lower than the MDS estimator. Also, when the standard deviation \( \sigma \) of the Gaussian noise is less than 100 \( \mu s \) the estimators are unbiased as shown in Fig. 6.

VI. CONCLUSIONS

In this work, a relative position system has been presented where only acoustic emissions are used as ranging method. The data are obtained in every node by using simultaneous round-trip-times-of-flight measurement of encoded acoustic emissions. After that, using the collected information, it is possible to obtain the position of every node using the multidimensional scaling technique. This algorithm allows to obtain a solution in the smallest number of dimensions, and it has a low computational cost. Nevertheless, signal processing delays perturb the measurement carried out with the proposed ranging method, by introducing an error in the estimation. Considering the nuisance parameters and using a non-linear least squares, a closer solution of the coordinates can be obtained.

Figure 5. 95% uncertainty ellipse for a distribution of nodes considering Gaussian noise in the measurement with variance \( \sigma^2 = 100 \mu s^2 \).

Figure 6. a) Total Variance on the estimation of the coordinates using the proposed positioning algorithms. b) Total bias in the estimation.

VII. ACKNOWLEDGMENT

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