Efficient Real-Time Correlator for LS Sequences

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Abstract—The cross-correlation function and the side-lobes of the auto-correlation function of LS codes are zero in a certain vicinity of the zero shift. Therefore, the effects of the Inter-Symbol-Interference and Multiple-Access-Interference that appears in CDMA and multi-sensor systems are mitigated. Conventionally, the detection of these sequences has been achieved by means of straight-forward matched-filter correlators. In this paper, a new correlator which significantly reduces the total number of operations to be performed is proposed, allowing real-time operation.

I. INTRODUCTION

Pulse compression sensor systems and CDMA (Code Division Multiple Access) systems are interference limited. There exists Inter-Symbol-Interference (ISI) due to the non zero auto-correlation (AC) sidelobes of the used sequences. They also have Multiple-Access-Interference (MAI) due to the non zero cross-correlations (CC) values. To avoid the effects of both ISI and MAI, the sidelobes of the AC and the CC values should be as small as possible.

Hence, a lot of research efforts have been devoted to finding sequences which fulfill both conditions as far as possible. Barker codes [2] have been widely used due to their good aperiodic AC. However, their maximal length is limited, so it is not possible to detect them in case of high noise. Furthermore, there are no Barker codes with low CC among them. Pseudorandom sequences such as m-sequences, Gold codes or Kasami codes [3], exhibit non zero off-peak AC and CC values in the case of asynchronous transmissions. Another possibility is Golay pairs [4], which have good aperiodic correlation properties, and their length is not limited. The detection of Golay sequences can be performed by means of efficient correlation algorithms which notably decrease the computational load and hardware complexity [5]. Nevertheless, they provide only two mutually orthogonal pairs, which is not useful for multi-user environments. Complementary sets of \(M\) sequences (\(M\)-CSS) [6] are a generalization of Golay codes containing more than two sequences. The elimination of the constraint in the number of sequences of a set yields on a high process gain and also \(M\) mutually orthogonal sets. To achieve these goals, it is necessary to add the AC functions of the sequences of the set, or the CC functions of the corresponding sequences in the \(M\) sets, respectively. The problem is that, in many systems, transducers have limited bandwidths, so it is not possible to transmit or receive the \(M\) sequences of the set simultaneously. Unfortunately, it implies an undesired increase of both ISI and MAI.

Loosely Synchronized (LS) codes [7] exhibit an Interference Free Window (IFW), where the aperiodic AC sidelobes and CC values become zero. Consequently, ISI and MAI are completely reduced if the maximum transmission delay is less than the length of the IFW. Usually, the correlation of these codes is carried out by means of straightforward matched filter implementations [8]. These implementations provide large processing cost; and, in case of a large amount of data to be processed, real-time operation can not be possible without the aid of high complexity hardware.

The contribution of this paper is to propose an Efficient LS correlator (ELSC), which significantly reduces the number of operations performed, in comparison with an straightforward matched filter implementation. The presented ELSC is designed for the correlation of LS codes, generated from Golay complementary pairs, as it is explained in [7]. Also, the practical implementation of the ELSC in a Field Programmable Gate Arrays (FPGA) has been carried out.

The paper is organized as follows. Section II introduces the family of LS codes. In Section III the Efficient Correlator of LS sequences is presented. Section IV shows the hardware implementation of the correlator and some results. In Section V tests with real signals are considered. Finally, conclusions are outlined in Section VI.

II. LS CODES

In [7], a systematic method to construct LS codes by using Golay complementary sequences is presented. It basically consists of using two orthogonal Golay pairs of length \(N\), and of linking the sequences of these pairs directly, or negated, depending on the coefficients of a \((P \times P)\) Hadamard matrix. Also, a set of \(W_0\) zeros has to be inserted in the centre of the LS code \(G_k\). The total length of the LS codes is given by \(L = KN + W_0\), where \(K = 2P\) is the number of available codes with orthogonal properties in the IFW (with \(K = 2^n\), \(n \in \mathbb{N}\)). The correlation properties of LS sequences \((G_k = [g_{k,1}, g_{k,2}, \ldots , g_{k,L}]; 1 \leq k \leq K)\) are given by (1). It can be noted that the AC function is obtained when \(n_1 = n_2\), whereas the CC function when \(n_1 \neq n_2\).
Prior to describe the proposed ELSC, it is necessary to give more details about the generation method of LS codes presented in [7]. Also, the notation $LS(N,P,W_0)$ mentioned in [9] has been adopted.

Firstly, two orthogonal Golay pairs $(c_0,s_0)$ and $(c_1,s_1)$ of length $N$ are defined. The AC and CC properties of these pairs should satisfy (2):

$$R_{c_1,c_2}(m) + R_{s_1,s_2}(m) = \begin{cases} 
2 \cdot N, & \text{for } m = 0, i = j \\
0, & \text{for } m \neq 0, i = j \\
0, & \text{for } \forall m, i \neq j
\end{cases}$$

Golay pairs $(c_0,s_0)$ and $(c_1,s_1)$ are linked depending on the coefficients of a Hadamard matrix $H$, and on a vector $\pi$. $H$ is a $P \times P$ Hadamard matrix, whose $h_{i,j} = \pm 1$ elements determine the final polarity of the Golay pairs. The vector $\pi = [\pi_1, \ldots, \pi_P]$, $\pi_k \in \{0, 1\}$ denotes a binary expansion of an arbitrary integer $n$, $0 \leq n < 2^P$ so that $n = \sum_i \pi_i \cdot 2^i$. This vector $\pi$ indicates which Golay pair, $(c_0,s_0)$ or $(c_1,s_1)$, is taken into account at every moment. Furthermore, a second vector $\pi^* = [\pi_1^*, \ldots, \pi_P^*]$, $\pi_k^* = \pi_k + 1 (mod 2)$ for each $1 \leq k \leq P$, has to be considered. The first $P$ sequences of the set $G_1, \ldots, G_P$ are constructed as it is shown in (3). Whereas the next $G_{P+1}, \ldots, G_K$ are obtained when the vector $\pi$ in (3) is substituted by its complement $\pi^*$. For brevity, polynomial notation has been used. In this notation, any bipolar sequence $a = [a_0, a_1, \ldots, a_{N-1}]$, is simply replaced by the polynomial $A(z) = \sum_{n=0}^{N-1} a_n z^n$, in the Laurent Polynomial ring $\mathbb{Z}[z,z^{-1}]$.

$$G_k(z) = \sum_{i=1}^{P} h_{k,i}[C_{\pi_i}(z) + z^{PN+W_0}S_{\pi_i}(z)]z^{(i-1)N}$$

Thus, a set of $K$ LS codes with length $L = KN + W_0$ is obtained. For periodic emissions it is necessary to insert a guard interval of, at least, $W$ is obtained. For periodic emissions it is necessary to insert a guard interval of length $W$. These cases $L = KN + 2W_0$. The LS codes generated exhibit an IFW of length $W = min \{2N - 1, 2W_0 + 1\}$, considering $W_0 = N - 1$ in most of cases [7][9].

Fig. 1 shows the aperiodic AC of a $LS(16,2,15)$ sequence; whereas in Fig. 2 can be seen the aperiodic CC among different $LS(16,2,15)$ codes from the same set.

III. EFFICIENT LS CORRELATOR

When longer LS sequences have to be processed in real-time, the increasing in the computational load demands the use of efficient correlators, able to perform the detection of these sequences with affordable computational cost. In this paper, an efficient correlator for LS sequences (ELSC) has been developed; which decreases the total number of operations performed, in comparison with a straightforward matched filter implementation. This correlator can be easily implemented on reconfigurable hardware to achieve real-time operation.

The proposed ELSC correlator exploits the properties of the Golay codes to simplify the correlation process. Golay pairs, $(c_0,s_0)$ and $(c_1,s_1)$, that form the LS sequences can be correlated by means of the Efficient Golay Correlator (EGC) developed in [5]. Later, the corresponding outputs of the EGCs are delayed and added or subtracted, depending on the values of the vector $\pi$ and the Hadamard matrix $H$. The block diagram of Fig. 3 shows the EGC algorithm structure. It is composed by a set of $M$ similar stages, where $N = 2^M$, with $M \in \mathbb{N} - \{0\}$, is the length of the Golay codes.
pairs. Every stage is composed of a delay module $z^{D_i}$, where $D_i = 2^{P_i}$, and $P_s$ is any permutation of the numbers $0, 1, \ldots, M - 1$; a multiplier $u_i$, where $U = [u_0, u_1, \ldots, u_{M - 1}]$ is the generation seed of the Golay pair; an adder; and a subtracter. It is important to note that, since the coefficients $u_i$ are binary, multiplications are reduced to additions and subtractions. Starting from any input signal $r[m]$, the EGC provides two outputs, $R_{r,c_1}[m]$ and $R_{r,s_1}[m]$, which are the correlation between the input signal $r[m]$, and the sequences of the Golay pair $(c_1, s_1)$. The number of multiplications in the EGC is equal to $\log_2 N$, whereas in the straightforward matched filter implementation it would be $N$. Also, the number of additions is reduced from the $N - 1$ operations in the straightforward to $2 \cdot \log_2 N$ in the EGC.

Considering the EGC, the scheme has to be adapted to the features of LS codes. Since two orthogonal pairs, $(c_0, s_0)$ and $(c_1, s_1)$, form the LS codes, two EGC are needed. One EGC carries out the correlation among the input sequence $r[m]$ and the Golay pair $(c_0, s_0)$, generated with seed $U_0$. It provides the outputs $R_{r,c_0}[m]$ and $R_{r,s_0}[m]$. The other EGC performs the correlation with the Golay pair $(c_1, s_1)$, generated with seed $U_1$, and gives the outputs $R_{r,c_1}[m]$ and $R_{r,s_1}[m]$.

Vectors $\pi$ or $\pi^*$ used to generate the LS sequence determine which correlation outputs, $R_{r,c_0}[m]$, $R_{r,s_0}[m]$, $R_{r,c_1}[m]$ or $R_{r,s_1}[m]$, have to be delayed. Note that the order of the delays in (3) has to be interchanged for the correlation. Therefore, the output corresponding with the correlation between the input signal and the sequences of the Golay pairs that firstly appear in the LS code, has to be delayed by $z^{P_N + W_0 + (P - 1)N}$. Later, the delayed outputs are added or subtracted, according to the column of the Hadamard matrix used in the LS code generation. Eq. (4) summarizes the operations performed by the ELSC. Furthermore, Fig. 4 depicts the block diagram of the ELSC.

$$R_{G_k} = \sum_{i=1}^{P} h_{k,i} z^{P_N + W_0} R_{C_{\pi_i}}(z) + R_{S_{\pi_i}}(z) z^{(P - i)N}$$

This optimization presents some advantages compared to the implementation of a classical straight-forward correlation. Table I shows the operations carried out by the two correlation schemes. As has been stated before, multiplications are reduced to additions and subtractions, due to the binary nature of their coefficients. Regarding the notation, $DW$ is the number of bits of the input signal; and $O_s = \frac{acquisition\ frequency}{emission\ frequency}$ is the over-sampling factor used when the sequences are acquired. It can be observed that the number of operations is significantly reduced in the case of the ELSC implementation. Nevertheless, the total number of memory bits to store the data in the ELSC, is larger than the corresponding in the straightforward implementation, as can be seen in Table II. To expand further, Fig. 5 and Fig. 6 depict the information given by Tables I and II respectively. To generate both figures, a set of LS codes with $K = 16$ sequences has been used, with $DW = 8$, and $O_s = 10$. The length $N$ of the initial Golay sequences can take values $N = [8, 16, 32, 64, 128, 256, 512, 1024]$; and the number of zeros in the centre of the LS code is $W_o = N - 1$.

IV. ELSC IMPLEMENTATION

A. Design Strategy

A generic hardware implementation in a FPGA of the ELSC has been developed. The correlator parameters can be changed through synthesizing the design. Therefore, it is possible to configure the number of LS codes availables in the set through

<table>
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<tr>
<th>TABLE I</th>
<th>OPERATIONS TO PERFORM FOR THE CORRELATION OF LS CODES, USING AN STRAIGHTFORWARD IMPLEMENTATION AND AN ELSC.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implementation</td>
<td>Multiplications</td>
</tr>
<tr>
<td>STRAIGHT-FORWARD</td>
<td>$L = KN + W_0$</td>
</tr>
<tr>
<td>ELSC</td>
<td>$2\log_2 N + K$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>MEMORY REQUIRED TO STORE THE DATA IN THE CORRELATION OF LS CODES, USING AN STRAIGHTFORWARD IMPLEMENTATION AND AN ELSC.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implementation</td>
<td>Memory requirements</td>
</tr>
<tr>
<td>STRAIGHT-FORWARD</td>
<td>$L \cdot O_s \cdot DW + L \cdot (DW + \log_2 L)$</td>
</tr>
<tr>
<td>ELSC</td>
<td>$2 \cdot O_s \cdot (\frac{DW}{2} + (DW + 1) \cdot \frac{DW}{2} + (DW + 2) \cdot \frac{DW}{2} + \ldots + (DW + \log_2 N - 1)) + O_s \cdot (DW + \log_2 N - 1) \cdot \frac{DW}{2} + N \cdot W_0) \cdot \frac{DW}{2} + \sum_{i=1}^{\frac{DW}{2}} \left(2 \cdot N \cdot i\right)$</td>
</tr>
</tbody>
</table>

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Memory required to store the data in the correlation of LS codes

Fig. 6. Memory versus \( N \) performance comparison of an ELSC and a straight-forward matched filter implementation of a correlator for LS codes. \( K = 16 \) users are supported, with \( DW = 8 \) and \( O_s = 10 \).

Operations to perform for the correlation of LS codes

Fig. 5. Operations versus \( N \) performance comparison of an ELSC and a straight-forward matched filter implementation of a correlator for LS codes. \( K = 16 \) users are supported, with \( DW = 8 \) and \( O_s = 10 \).

The parameter \( n \) (\( K = 2^n \)). The number of zeros \( W_0 \) inserted in the centre of the LS code can be selected as well. It is possible to configure the number \( M \) of stages in the EGC, and therefore the length \( N = 2^M \) of the initial Golay codes. The data-width \( DW \) can be selected, in order to have more accuracy in the obtained results. Finally, the over-sampling rate \( O_s \) can be also selected by the user.

Fig. 7 shows the structure of the hierarchical root block for the ELSC, where \( r[m] \) is the input signal; \( U_0 \) and \( U_1 \) are the generation seeds of the Golay pairs; \( P_i \) and \( H_{ad} \) are the vector \( \pi \) and the column of the Hadamard matrix \( H \) used in the generation of the LS codes; and finally \( R_{r,G_k} \) is the correlation function between the input signal \( r[m] \) and the considered LS sequence \( G_k \). Note that the output \( R_{r,G_k} \) requires \( DW + M + n \) bits if overflow is avoided in internal operations.

The internal architecture of the ELSC is illustrated in Fig. 8. It is based on six different modules. The first one implements the EGC (see Fig. 3), and it is based on \( M \) similar stages. As has been mentioned before, the number of stages is configurable in order to increase the adaptability of the system. Every stage is divided into two modules: a sequential and a combinational one. The sequential block contains the specific delay of the stage. Note that, at every stage, the number of bits required to store the partial results is increased. Hence, to reduce the total memory required by the correlator, the larger delays should be placed at the first stages, and the smaller ones at the last stages. On the other hand, all these delay elements have been increased by \( O_s \), to consider the sampling factor used in the acquisition of the input signal. As a result, \( D = [D_1, D_2, \cdots, D_M] = [2^{M-1} \cdot O_s, 2^{M-2} \cdot O_s, \cdots, 2^0 \cdot O_s] \).

These delays have been implemented by making use of specific shift registers in Xilinx FPGA’s architecture (SRL16 modules [10]). The combinational module carries out the operations of the stage. The binary values of the seeds \( U_0 \) and \( U_1 \) determine the final configuration of the adders and subtractors in the \( i \)-th stage.

The second block, called MUX, generates a set of \( P \) multiplexers governed by the input \( P_i \). Every multiplexer has the same two inputs, \( a \) and \( b \); which can be connected either with \( (R_{r,G0}, R_{r,G1}) \), or with \( (R_{r,G0}, R_{r,G1}) \). If \( P_i(i) = '0' \) then the multiplexer output is \( a \), whereas if \( P_i(i) = '1' \) the multiplexer output is \( b \).

When the outputs of the EGCs are selected, they have to be delayed according to (4). There are two different delays blocks: one implements \( P \) delays, all of them with value \( z^{(P \cdot N + W_0)} \); the other contains the \( [z^0, z^1, \cdots, z^{(P-1) \cdot N}] \) delays. Both blocks use the basic module SRL16 of Xilinx FPGA’s architectures.

The Hadamard block determines the polarity of the delayed EGC’s outputs. \( H_{ad}(i) = '0' \) denotes negating the input sequence \( i \); while \( H_{ad}(i) = '1' \) means that the output sequence \( i \) and the input sequence \( i \) are both equal.

Finally, the \( P \) outputs of both Hadamard blocks are added
to obtain the correlation output $R_{r,G_k}$.

### B. Performance of the Hardware Implementation

The result requirements and maximum frequencies in the ELSC implementation depend on the $(N, P, W_0)$ parameters of the LS code. In Table III the resource requirements of $LS(8,2,7)$, $LS(16,2,15)$ and $LS(8,4,7)$ are shown. An over-sampling factor $O_s = 10$ has been used, with a data-width $DW = 8$. The correlation system has been implemented in a Spartan3 xc3s1500 FPGA by Xilinx [10].

A performance comparison between the ELSC and a straight-forward matched filter implementation has been achieved. Fig. 9 depicts the hardware implementation scheme of a straightforward correlator based on internal BRAM memory [11]. The input signal is digitalized with a sampling frequency $f_s$, and stored in a buffer of size $L \cdot O_s$, that has been implemented in BRAM blocks [10]. This sampling buffer is read every $f_{FPGA} = L \cdot f_s$. At each reading, a sample is added or subtracted with the previous accumulated result, depending on the LS code evaluated. Whenever a new sample is received, the accumulated value is reset in order to compute a new correlation. The access to the sampling buffer is carried out in gaps of $O_s$ positions, by taking into account the sampling frequency. In Table IV the requirements for this implementation are shown, in case of using a $LS(16,2,15)$. Correlation results are obtained every $t_{s} = f_{FPGA} \cdot L = 8.01 \text{ ns} \cdot 79 = 632.95 \text{ ns}$, being $t_s = \frac{1}{f_s}$ and $t_{FPGA} = \frac{f_s}{f_{FPGA}}$. Nevertheless, the ELSC implementation provides a new correlation result every FPGA clock cycle $t_{FPGA} = t_s = 10.23 \text{ ns}$, assuring real-time operation, although it requires more resources.

To increase the operation frequency of the straight-forward correlator, tasks have to be overlapped temporarily, which implies introducing registers to store intermediate data. Therefore, the amount of required resources increases, approaching to the required by the ELSC implementation. Thus, it can be stated that the ELSC implementation is more suitable for real-time correlation of LS sequences.

![Fig. 8. Internal architecture of the ELSC.](image)

![Fig. 9. Sequential implementation of a straight-forward correlator based on internal BRAM memory.](image)

### Table III

<table>
<thead>
<tr>
<th>xc3s1500</th>
<th>$LS(8,2,7)$</th>
<th>$LS(16,2,15)$</th>
<th>$LS(8,4,7)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slices</td>
<td>720</td>
<td>1369</td>
<td>2112</td>
</tr>
<tr>
<td>Luts</td>
<td>929</td>
<td>1636</td>
<td>2432</td>
</tr>
<tr>
<td>IOBs</td>
<td>33</td>
<td>36</td>
<td>38</td>
</tr>
<tr>
<td>Max. Freq. $f_s = f_{FPGA}$</td>
<td>116.550 MHz</td>
<td>97.771 MHz</td>
<td>84.374 MHz</td>
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<tr>
<td>Correlation time $t_s = f_{FPGA}$</td>
<td>8.38 ns</td>
<td>10.23 ns</td>
<td>11.85 ns</td>
</tr>
</tbody>
</table>

### Table IV

<table>
<thead>
<tr>
<th>xc3s1500</th>
<th>$LS(16,2,15)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slices</td>
<td>51</td>
</tr>
<tr>
<td>Luts</td>
<td>83</td>
</tr>
<tr>
<td>IOBs</td>
<td>27</td>
</tr>
<tr>
<td>Max. Freq. $f_{FPGA}$</td>
<td>124.813 MHz</td>
</tr>
<tr>
<td>Max. Freq. $f_s$</td>
<td>1.58 MHz</td>
</tr>
<tr>
<td>Correlation time $t_s$</td>
<td>632.95 ns</td>
</tr>
</tbody>
</table>
V. APPLICATION EXAMPLE

The suitability of the ELSC has been tested in the ultrasonic pulse compression system shown in Figure 10. A set of two LS codes \( L_S(64, 1, 63) \) is generated, although only one sequence \( L_S^1 \) is transmitted. A REGAL-RH16E [12] high power speaker has been used to transmit \( L_S^1 \) with a BPSK modulation scheme, which employs a symbol with one period of a 25 kHz squared signal. Two echoes (echo 1 and echo 2) of \( L_S^1 \) are received, due to the multipath caused by a reflector. These echoes are captured by a condenser microphone capsule Avisoft Bioacoustic CM16 [13], amplified and digitalized at a sampling frequency of 500 kHz, so \( O_z = 20 \). Afterwards, the resulting signal is demodulated and, by using two ELSCs, it is correlated with the two original sequences \( L_S^1 \) and \( L_S^2 \) of the set. Figure 11 shows the results obtained in every ELSC. At the output of the correlator \( L_S^1 \) only echo 1 will be validated. Echo 2 arrives within the IFW of echo 1, and, due to the AC property, there is no interference impact in this zone. Also, because of the CC property, there is no interference at the output of the correlator \( L_S^2 \).

VI. CONCLUSIONS

Traditional sequences used in pulse compression and CDMA systems, present ISI and MAI. LS sequences have ideal aperiodic correlation properties in a small window around the zero shift, removing the ISI and MAI completely in this window. If real-time operation is a requirement of the application, traditional straight-forward matched filter implementations are unsuitable. In this paper, an efficient correlator for LS sequences, generated with the method detailed in [7], is presented. It notably decreases the total number of operations to carry out, achieving real-time operation.

A generic hardware implementation in FPGAs of the proposed LS correlator has been developed. The design is characterized by five generic parameters: the number of codes in the set; the number of ceros in the centre of the LS code; the length of the Golay pairs used to generate the LS codes; the width of the data-path; and the sampling factor. Hence, the result is a configurable module, able to be adapted to the requirements of the application. The performance of the correlator has been tested on a Xilinx Spartan3 FPGA, and compared with the results obtained with an straight-forward implementation. Tests have shown that, for real-time operation, the correlator presented in this paper is more adequate. Furthermore, it has been included on an ultrasonic sensory system, to verify the detection capability in case of working in a multipath environment.

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